Technical Notes

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Localized Linearization Method for Wings at High Angle of Attack

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Introduction

VARIOUS versions of the vortex lattice method (VLM) are widely used for wings at a high angle of attack since it is applicable to complex wing planforms and requires less computer time than other methods. Some of these versions use the Prandtl-Glauert transformation to account for the compressibility effects, the governing equation is linear, and nonlinearity is introduced through the edge separation. For wings at a high angle of attack and subcritical Mach number, the solution requires a nonlinear governing equation as well as a nonlinear boundary condition due to the edge separation. Hence, the problem may be seen as having nonlinearities in two aspects. Recently, Kandil[1] solved this problem by using an integral formulation with a volume distribution of sources to account for the nonlinear terms in the differential equation for the velocity potential. In this Note, a localized linearization method is developed to extend the use of the VLM for solving the same problem. This method has the advantages of the VLM without involving the flowfield discretization, and hence, it remains a surface boundary-element formulation.

Mathematical Formulation

The small perturbation assumption actually is not reasonable for separated flow at a high angle of attack. Many investigations showed that in separated flow, the spanwise perturbation velocity component is greater than the chordwise one and sometimes even attains a value nearly equal to half of the freestream velocity. Thus, it is necessary to include all nonlinear terms of the second order in the velocity potential equation as follows:

$$(1 - M_\infty^2)\phi_{xx} + \phi_{yy} + \phi_{zz} = \psi \tag{1}$$

and

$$\psi = M_\infty^2(\gamma + 1)\phi_x\phi_{xx}/U + M_\infty^2(\gamma - 1)\phi_x \times (\phi_{yy} + \phi_{zz})/U + 2M_\infty^2(\phi_y\phi_{xy} + \phi_z\phi_{xz}) \tag{2}$$

where ϕ is the perturbation potential. The M_∞ and U are the freestream Mach number and velocity, respectively, and γ is the ratio of specific heats. The subscript xx, yy, and zz mean second derivatives with respect to x, y, and z, repeatedly.

In the integral method of Kandil, Eq. (1) is treated as a Poisson equation for which the integral expression of the solution will comprise both surface and volume integrals. Therefore, the numerical computation will involve not only the wing surface but also a flowfield discretization. Here, a different approach is adopted. With some algebraic manipulation, Eqs. (1) and (2) are transformed into

$$(1 - M_S^2)\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \tag{3}$$

and

$$M_S^2 = \frac{M_\infty^2[1 + 2(\phi_x\phi_{xx} + \phi_y\phi_{xy} + \phi_z\phi_{xz})/U\phi_{xx}]}{1 - M_\infty^2(\gamma - 1)\phi_x/U} \tag{4}$$

where M_s is only a symbol.

When the local value of M_s is treated temporarily as a constant, Eq. (3) turns out to be linear in a localized sense and the nonlinear VLM can be applied to deal with the separated flow. The nonlinear VLM uses an iteration process to determine the position of the vortex shed from the wing edges; meanwhile at each step of iteration, M_s has to be modified according to Eq. (4). This single-cycle iteration procedure constitutes the localized linearization method of the present paper. Iterations will proceed until satisfactory convergence of both the separated vortex position and the M_s distribution are achieved.

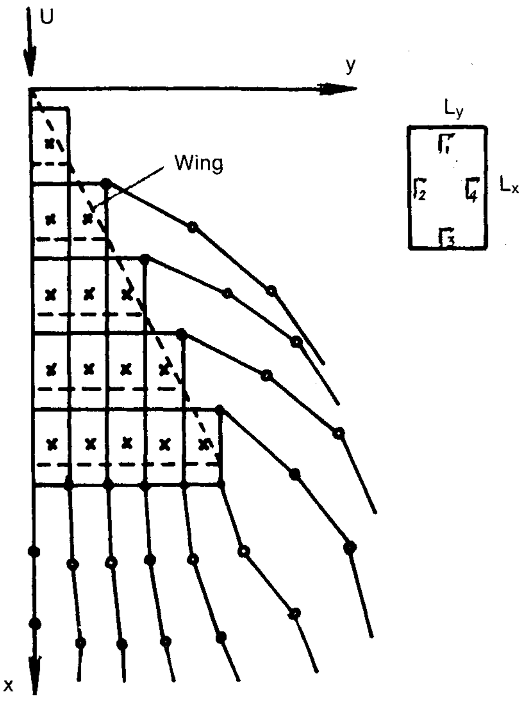

Fig. 1 Vortices on the wing with leading-edge separation.

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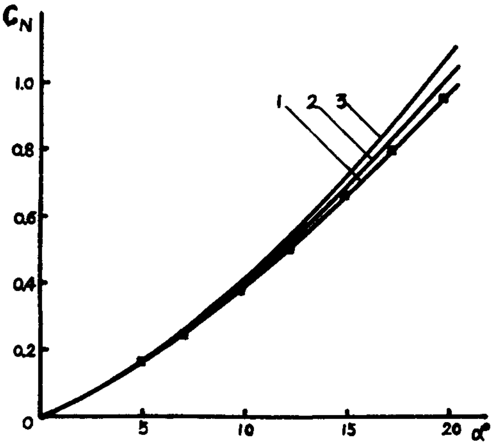

Fig. 2 Normal force coefficient vs angle of attack for a rectangular wing of $AR = 1$. Curves 1) $M_\infty = 0.1$, 2) $M_\infty = 0.5$, 3) $M_\infty = 0.7$. Experiment □: $M_\infty = 0.1$.

Nonlinear terms are also included in the formula for the coefficient of the pressure difference between upper and lower wing surfaces, i.e.,

$$\Delta C_P = \frac{-\left[2U\Delta\phi_x + \beta^2(\phi_{xU} + \phi_{xL})\Delta\phi_x + (\phi_{yU} + \phi_{yL})\Delta\phi_y\right]}{U^2} \tag{5}$$

where

$$\Delta\phi_x = \phi_{xU} - \phi_{xL}, \qquad \Delta\phi_y = \phi_{yU} - \phi_{yL}, \qquad \beta^2 = 1 - M_\infty^2$$

The subscripts U and L indicate upper and lower surfaces, respectively. Let

$$\phi_{xU} = \phi_{xm} + \frac{\Delta\phi_x}{2} \qquad \phi_{xL} = \phi_{xm} - \frac{\Delta\phi_x}{2}$$

$$\phi_{yU} = \phi_{ym} + \frac{\Delta\phi_y}{2} \qquad \phi_{yL} = \phi_{ym} - \frac{\Delta\phi_x}{2}$$

Then,

$$\Delta C_P = \frac{-(2U\Delta\phi_x + 2\beta^2\phi_{xm}\Delta\phi_x + 2\phi_{ym}\Delta\phi_y)}{U^2} \tag{6}$$

where ϕ_{xm} and ϕ_{ym} are the x and y direction perturbation velocity components of any point induced by all vortices, except the four vortex filaments surrounding the point under consideration. Also, $\Delta\phi_x$ and $\Delta\phi_y$ are the x and y direction tangential velocity component differences between the upper and lower wing surfaces. For the vortex lattice shown in Fig. 1,

$$\Delta\phi_x = \frac{(E\Gamma_1 + \Gamma_3)}{2L_x} \qquad \Delta\phi_y = \frac{(\Gamma_2 + \Gamma_4)}{2L_y} \tag{7}$$

where $E = 1$ for a normal lattice and 2 for a leading-edge lattice. The above-mentioned procedure for calculating the pressure coefficient follows the work of Konstadinopoulos.[2]

Having obtained the ΔC_P distribution, the resultant normal force and moment coefficients are

$$C_N = \frac{\Sigma\Delta C_{Pi}\Delta S_i}{\Sigma\Delta S_i} \qquad C_M = \frac{\Sigma\Delta C_{Pi}\Delta S_i(x_i - x_o)}{C\Sigma\Delta S_i}$$

Numerical Implementation

It is well known that the nonlinear VLM is sensitive to the shape of the divided lattice, especially for delta wings. In this Note, the separated vortex model shown in Fig. 1 is used. To assure convergence with a lattice mesh that is not dense, the computation format of M_s is taken to be

$$M_{s_{i+1}}^2 \Leftarrow 0.5M_{s_i}^2 + 0.5M_{s_{i+1}}^2$$

The second derivatives are obtained by finite differences as

$$\phi_{xx} = \frac{\Delta(\phi_x)}{\Delta x} \qquad \phi_{xy} = \frac{\Delta(\phi_y)}{\Delta x} \qquad \phi_{xz} = \frac{\Delta(\phi_z)}{\Delta x}$$

The first derivatives ϕ_x, ϕ_y, ϕ_z, which are the three components of the perturbation velocity, are the direct results of each step of calculation in any VLM program. Hence, in the present method although double sources of nonlinearities are taken into account, only a few extra computational steps are added.

Illustrative Examples and Discussion

Delta and rectangular wings of small aspect ratio at different Mach numbers are calculated by our method.

Figure 2 shows the results for a rectangular wing of $AR = 1$, having side-edge separation. The mesh of panels on a half-

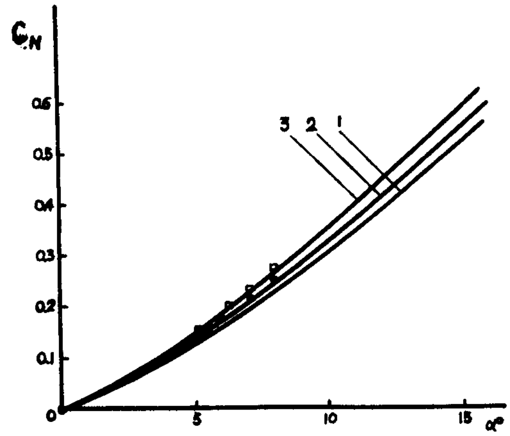

Fig. 3a Normal force coefficient vs angle of attack for a delta wing of $AR = 1.07$. Curves 1) $M_\infty = 0.0$, 2) $M_\infty = 0.6$, 3) $M_\infty = 0.8$. Experiment ○: $M_\infty = 0.8$.

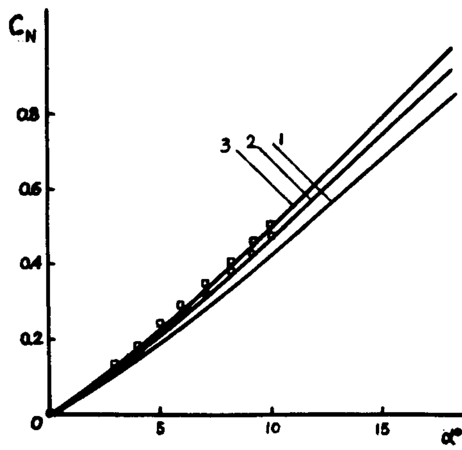

Fig. 3b Normal force coefficient vs angle of attack for a delta wing of $AR = 1.865$. Curves 1) $M_\infty = 0.0$, 2) $M_\infty = 0.7$, 3) $M_\infty = 0.85$. Experiment ○: $M_\infty = 0.7$, □: $M_\infty = 0.85$.

wing is 3×6. The results for delta wings of $AR = 1.07$ and 1.865 with leading-edge separation are presented in Fig. 3. The mesh on the half-wing is 5×5 and 6×6, respectively. Experimental data in both figures are obtained from Ref. 3. It can be seen that the results of the present method agree well with the experiment.

The calculated wing loads from the reduced linear basic equation are higher than the present results. This conclusion was also obtained by the integral method of Ref. 1.

From these examples, it is found that the nonlinear terms of the basic equation are as important for a moderate Mach number as a higher Mach number. And for these double sources of nonlinearity problem, the localized linearization method is a better approach in respect to convergence of computation. Besides, this method is much more economical as the number of panels used is considerably less than the integral method that involves additional three-dimensional space discretization.

References

[1]Kandil, O.A., "Computational Technique for Compressible Vortex Flows past Wings at Large Incidence," *Journal of Aircraft*, Vol. 22, Sept. 1985, pp. 750–755.

[2]Konstadinopoulos, P., Mook, D.T., and Nayfeh, A.H., "A Numerical Method for General Unsteady Aerodynamics," AIAA Paper 81-1877, 1981.

[3]Kandil, O.A., Mook, D.T., and Nayfeh, A.H., "Effect of Compressibility on the Nonlinear Prediction of the Aerodynamic Loads on Lifting Surfaces," AIAA Paper 75-121, 1975.

Application of the Boundary Element Method to the Thin Airfoil Theory

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I. Introduction

IN the conventional theory of the thin airfoils (see Ref. 1), the following two symplifying hypotheses are assumed: 1) the boundary condition is linearized and 2) this condition is transferred on the chord of the airfoil. In the theory presented here these hypotheses are given up (hence the exact condition is used on the airfoil, i.e., the natural setting).

In the literature there exists the paper by Hess and Smith[2] where the boundary element method (BEM) is used for nonlifting three-dimensional bodies in incompressible fluids, but these authors use the so-called indirect method[3] assimilating the body surface with a source distribution. The method used here differs from the Hess-Smith method in that the method used here is direct and does not assimilate the body with a source distribution. As it is known,[4] in this type of problem, the direct methods give better results than the indirect methods.

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The idea underlying BEM is to use the fundamental solutions of the equation of motion in order to reduce the boundary-value problem to an integral equation on the body boundary and then to solve this equation by discretization. In the present paper, we prefer to consider the fundamental quantities p and v and not the velocity potential as conventional because we intend to obtain and equation in p. The aerodynamics of interest are the pressure values on the wings and not the values of the potential.

II. Equations of Motion

We want to determine the perturbation produced in a uniform subsonic stream of velocity U_∞, pressure p_∞, and density ρ_∞ by an airfoil C. We use the reference frame x_1Oy_1 with the Ox_1 axis in the direction of the unperturbed stream and O at the leading edge. We introduce the dimensionless variables X, Y defined by relation $(x_1,y_1) = L_0(X,Y)$, L_0 being the length of the airfoil chord. Denoting V_1 the total velocity and P_1 the total pressure, we have

$$V_1 = U_\infty(1+V), \qquad P_1 = p_\infty + \rho_\infty U_\infty^2 P \tag{1}$$

V and P being the dimensionless perturbation velocity and pressure, respectively, determined by system[5]

$$M_\infty^2 \partial P/\partial X + \mathrm{Div}\ V = 0, \qquad \partial V/\partial X + \mathrm{Grad}\ P = 0 \tag{2}$$

and boundary condition

$$(1+V)\cdot N = 0 \text{ on } C \tag{3}$$

and damping condition

$$\lim\ (P,V) = 0 \tag{4}$$

We denote M_∞ (<1) the Mach number in the free flow and N the inner normal to C. With $V = (U,V)$ from Eq. (2) we deduce $U = -P$ and

$$\beta^2 \partial P/\partial X - \partial V/\partial Y = 0 \qquad \partial V/\partial X + \partial P/\partial Y = 0 \tag{5}$$

where $\beta = \sqrt{1-M_\infty^2}$

Performing the change of variable X, $Y \rightarrow x,y$

$$x = X, \qquad y = \beta Y \tag{6}$$

and the change of functions $P, V \rightarrow p, v$

$$p = \beta P, \qquad v = V \tag{7}$$

the system of Eq. (5) becomes

$$\partial p/\partial x - \partial v/\partial y = 0, \qquad \partial v/\partial x + \partial p/\partial y = 0 \tag{8}$$

and the boundary condition of Eq. (3) reads

$$pn_1 - vn_2 = \beta n_1 \tag{9}$$

and the damping condition becomes

$$\lim_{\infty} (p,v) = 0 \tag{10}$$

In Eq. (9) $n = (n_1,n_2)$ is the unit vector of the interior normal to the boundary C in the new variables.

III. Integral Equation

Using the Fourier transform method, it is shown that the solution of system

$$\partial p^*/\partial x - \partial v^*/\partial y = \delta(x-\xi), \quad \partial v^*/\partial x + \partial p^*/\partial y = 0 \tag{11}$$